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## **AZ SOLAR ENERGY<sup>1</sup>**

Joe Light wants to be an AZ state senator, transforming AZ into the solar center of the world. The greatest natural resource of AZ, he thought, is the sun (shines some 300 days a year), which invites residents from the Midwest, Northeast and Canada to enjoy the wonder weather during the North American winter. A politically attractive use of this great resource is in generating electricity, without polluting the planet. Currently the technology is such that a government subsidy is required to compete with coal, or even natural gas. Joe wondered what type of subsidy he should advocate, which would please the voters and incentivize the adoption of solar.

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<sup>1</sup> © Dean A. Paxson (Manchester Business School). This case is for the purpose of class discussion only and is not intended as an illustration of either good or bad business practices or politics. The character of Joe Light is fictitious.

Do permanent or retractable government subsidies such as direct payments per unit revenue or freedom from taxation, encourage early investment in solar energy facilities? Does the size of the possible government subsidy reduce the revenue threshold that justifies investment significantly, even if the subsidy might be retracted?

The IPCC and indeed the BIS have suggested that to incentivize renewable energy investments, tax reliefs, premium payments, and guaranteed purchases might be required.”<sup>2</sup> Joe wonders whether altering any of the some six inputs for a renewable facility evaluation model could lower significantly the revenue that justifies making the investment, at a reasonable cost to the government.

Consider that the instantaneous revenue from a facility is the respective commodity price of the output times the quantity produced. For solar there is minimal operating cost, since sunshine is free in AZ. Assumptions are that the lifetime of the facility is infinite, there are no taxes on the facility income, or competition, and facility construction is instantaneous.

The next section considers a menu of possible arrangements, that is some characteristic subsidies for such facilities, first where there is no subsidy (Model 1); then assuming there is a permanent subsidy proportional to the revenue (Model 2); finally assuming there is a retractable subsidy proportional to the revenue (Model 3), as suggested in the Adkins and Paxson (2014), Appendix.

### ***Stochastic Revenue Models***

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<sup>2</sup> IPCC (2014), Chapter 7 page 72, and adds that “those that avoid unnecessary risks in project revenues are more effective.” BIS (2014), page 116, notes that “a key impediment ...is uncertainty about the pipeline of projects”.

Consider a perpetual opportunity to construct a small electricity generating facility producing  $Q$  MWhrs/pa, using solar power, at a fixed investment cost  $K$ . This investment cost is treated as irreversible or irrecoverable once incurred. The value of this investment opportunity, denoted by  $ROV$ , depends on the amount of output, and the price per unit of output, denoted by  $P^3$ ,  $P^*Q=R$ , revenue.  $R$  is assumed to be stochastic and to follow a geometric Brownian motion process:

$$dR = \alpha_R R dt + \sigma_R R dZ \quad (1)$$

where  $\alpha_R$  denotes the instantaneous drift parameter,  $\sigma_R$  the instantaneous volatility, and  $dZ$  the standard Wiener process. Assuming risk neutrality, the differential equation representing the value to invest for an inactive person (perhaps now using electricity from a utility supplier) with an appropriate investment opportunity (based on residing in AZ, or perhaps approval for the facility or a concession for infrastructure) is:

$$\frac{1}{2} \sigma_R^2 R^2 \frac{\partial^2 ROV_1}{\partial R^2} + \theta_R R \frac{\partial ROV_1}{\partial R} - r ROV_1 = 0. \quad (2)$$

where  $\theta_R$  denotes the risk-neutral drift rates ( $\alpha_R$  in this case) and  $r$  the risk-free rate. Adkins and Paxson (2015) show that the solution to (2) is:

$$ROV_1 = B_1 R^{\beta_1}. \quad (3)$$

$\beta_1$  is the power parameter for this option value function. Since there is an incentive to invest when  $R$  is sufficiently high but a disincentive when sufficiently low, the power parameter value is positive. Also, the power parameter is determined using the characteristic root equation (which is the positive root of a simple quadratic equation) found by substituting (3) in (2):

$$Q(\beta_1) = \frac{1}{2} \sigma_R^2 \beta_1 (\beta_1 - 1) + \theta_R \beta_1 - r = 0. \quad (4)$$

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<sup>3</sup> Output could be electricity or directly useful energy (like heat), stated here as the electricity use avoided.

After the investment, the solar plant generates revenue equaling  $(1+\tau)R$ , where  $\tau$  is the permanent subsidy proportional to the revenue sold ( $\tau=0$  indicates no possible subsidy). So from (2), the valuation relationship for the operational state is:

$$\frac{1}{2}\sigma_R^2 R^2 \frac{\partial^2 ROV_1}{\partial R^2} + \theta_R R \frac{\partial ROV_1}{\partial R} + (1+\tau)R - rROV_1 = 0. \quad (5)$$

After the investment (K), the solution to (5) is:

$$\frac{(1+\tau)R}{r-\theta_R}.$$

### **Model 1**

The subsidy is set to equal zero in Model 1. If the threshold revenue signaling an optimal investment is denoted by  $\hat{R}_1$ , then:

$$\hat{R}_1 = \frac{\beta_1}{\beta_1 - 1} K (r - \theta_R). \quad (A1)$$

The value for the investment opportunity is defined by:

$$ROV_1 = \begin{cases} B_1 R^{\beta_1} & \text{for } R < \hat{R}_1, \\ \frac{R}{r - \theta_R} - K & \text{for } R \geq \hat{R}_1. \end{cases} \quad (A2)$$

where:

$$B_1 = \frac{\hat{R}_1^{1-\beta_1}}{\beta_1 (r - \theta_R)}. \quad (A3)$$

### **Model 2**

For a positive proportional permanent subsidy  $\tau_M$ , the corresponding results are:

$$\hat{R}_2 = \frac{\beta_1}{\beta_1 - 1} K \frac{(r - \theta_R)}{(1 + \tau_M)}, \quad (\text{A4})$$

$$ROV_2 = \begin{cases} B_2 R^{\beta_1} & \text{for } R < \hat{R}_2, \\ \frac{R(1 + \tau_M)}{r - \theta_R} - K & \text{for } R \geq \hat{R}_2, \end{cases} \quad (\text{A5})$$

$$B_2 = \frac{(1 + \tau_M) \hat{R}_2^{1 - \beta_1}}{\beta_1 (r - \theta_R)} \quad (\text{A6})$$

### Model 3

The probability of a sudden unexpected withdrawal of the subsidy is denoted by  $\lambda$ . If the revenue threshold signaling an optimal investment is denoted by  $\hat{R}_3$ , then its solution is found

implicitly from:

$$\hat{R}_3 = \frac{\beta_3}{\beta_3 - 1} K \frac{r - \theta_R}{1 + (1 - \lambda) \tau_M} + B_1 \hat{R}_3^{\beta_1} \frac{\beta_3 - \beta_1}{\beta_3 - 1} \quad (\text{A7})$$

where  $B_1$  is from (A3). The value for the investment opportunity is specified by:

$$ROV_3 = \begin{cases} B_3 R^{\beta_3} + B_1 R^{\beta_1} & \text{for } R < \hat{R}_3, \\ \frac{R(1 + (1 - \lambda) \tau_M)}{r - \theta_R} - K & \text{for } R \geq \hat{R}_3, \end{cases} \quad (\text{A8})$$

where:

$$B_3 = \frac{(1 + (1 - \lambda) \tau_M) \hat{R}_3^{1 - \beta_3}}{\beta_3 (r - \theta_R)} - \frac{\beta_1}{\beta_3} B_1 \hat{R}_3^{\beta_1 - \beta_3}. \quad (\text{A9})$$

$\beta_3$  is the positive root of (4) with  $\lambda$  added to  $r$ . For  $\lambda = 0$ , when there is no likelihood of the subsidy being withdrawn unexpectedly,  $\beta_3 = \beta_1$  and Model 3 simplifies to the Model 2 solution.

It is easy to put these formulae into Excel as shown in Figures 1, 2, 3 below.

Figure 1

	A	B	C	D
1	<b>REVENUE MODEL 1</b>			
2	INPUT	Stochastic R		
3	P	10.00 Per MWhr		
4	Q	25.00 MWhrs/per annum		
5	R	250.00 B3*B4		
6	K	4867.00 Per Capacity of 75 MWhrs/per annum		
7	$\sigma$	0.05 Template		
8	r	0.10 Given		
9	$\theta$	0.04 Template		
10	$\tau$	0.00 NO SUBSIDY		
11	r- $\theta$	0.06 B8-B9		
12	$\lambda$	0.00 Probability		
13	OUTPUT			
14	ROV1	4.51 IF(B5<B18,B17*(B5^B16),B15)		A2
15	V-K	-700.33 ((1+B10)*B5/B11)-B6		
16	$\beta_1$	1.64		EQ 4
17	B1	0.000514 (B18^(1-B16))/B16*B11		A3
18	R*	745.06 B6*B11*(B16/(B16-1))		A1
19	$\beta_1$	(1/B7^2)*(-(B11-0.5*(B7^2))+SQRT((B11-0.5*(B7^2))^2+(2*B8)*(B7^2)))		

Figure 2

	A	B	C	D
1	<b>REVENUE MODEL 2</b>			
2	INPUT	Stochastic R		
3	P	10.00		
4	Q	25.00		
5	R	250.00 B3*B4		
6	K	4867.00		
7	$\sigma$	0.05		
8	r	0.10		
9	$\theta$	0.04		
10	$\tau$	0.20		
11	r- $\theta$	0.06 B8-B9		
12	$\lambda$	0.00 Probability		
13	OUTPUT			
14	ROV2	6.09 IF(B5<B18,B17*(B5^B16),B15)		A5
15	V-K	133.00 ((1+B10)*B5/B11)-B6		
16	$\beta_1$	1.64		EQ 4
17	B2	0.000693 ((1+B10)*B18^(1-B16))/B16*B11		A6
18	R*	620.88 (B6*B11/(1+B10))*(B16/(B16-1))		A4
19	$\beta_1$	(1/B7^2)*(-(B11-0.5*(B7^2))+SQRT((B11-0.5*(B7^2))^2+(2*B8)*(B7^2)))		

Figure 3

	A	B	C	D	E
1	<b>REVENUE MODEL 3</b>				
2	INPUT	Stochastic R			
3	P	10.00			
4	Q	25.00			
5	R	250.00 B3*B4			
6	K	4867.00			
7	$\sigma$	0.05			
8	r	0.10			
9	$\theta$	0.04			
10	$\tau$	0.20			
11	r- $\theta$	0.06 B8-B9			
12	$\lambda$	0.10 Probability			
13	OUTPUT				
14	ROV3	5.63	IF(B5<B18,B17*(B5^B16)+B24*(B5^B23),B15)		A8
15	V-K	49.67	((1+(1-B12)*B10)*B5/B11)-B6		
16	$\beta_3$	3.19			
17	B3	0.00000003	A9		
18	R*3	<b>366.55</b>			
19	Solver	0.0000	Set B19=0, Changing B18		A7
20	$\beta_1$	1.6446			
21	B1	0.000514			
22	R*1	745.06			
23	$\beta_3$	$(1/B7^2)*(-(B11-0.5*(B7^2))+SQRT((B11-0.5*(B7^2))^2+(2*(B8+B12))*(B7^2)))$			
24	R*3	$(B6*B11/(1+(1-B12)*B10))*(B16/(B16-1))+B24*(B18^B23)*((B16-B23)/(B16-1))-B18$			
25	B3	$((1+(1-B12)*B10)*B18^(1-B16))/B16*B11-(B23/B16)*B24*(B18^(B23-B16))$			

While some of the inputs are hypothetical (template), R and K are consistent with a small residential facility more than adequate for a family resident in AZ, with a capacity of 75 KWhrs/pa operating at around one-third load factor. Joe believes that at the current price of electricity in AZ, no one understanding real option theory and practice would currently install a facility with the specified capacity at the current investment cost, see Table 1. There are six policy ways to encourage early investment, by increasing the current expected revenue or decreasing the threshold the revenue that justifies immediate investment. (1) Increasing P or Q is similar to (2) increasing  $\tau$ , the proportional subsidy on R Adkins and Paxson (2015) imply that (3) reducing R volatility (see the AZ electricity price series in the Excel sheet “Prices”) would

reduce the R threshold, (4) surprisingly increasing  $\lambda$  reduces the R threshold in Model 3 and (5) reducing r reduces all R thresholds. Adkins and Paxson (2015) do not consider the implications of (6) reducing K, through direct investment cost reduction or indirectly through an investment tax credit. If a high rate (40%) taxpayer owns the facility, the effective investment cost is  $K(1-0.4 \cdot 0.3)$ . But altering any of these six policy variables is likely to have an impact on the ROV. Now do AZ voters really value the opportunities to invest in solar? Should the government seek to recoup some of this ROV through selling solar energy permits, which might compensate for some of the value of any subsidy? Table 1

**Subsidy Incentive Effect under Different Models**

	$\tau$	$R^\wedge$	$P^\wedge(Q^\wedge=25)$	ROV	
Model 1	0.00	745.06	29.80	4.51	NO SUBSIDY
Model 2	0.20	620.88	24.84	6.09	PERMANENT SUBSIDY
Subsidy Cost $R^\wedge 2$	\$124.18				
Model 3	0.20	366.55	14.66	5.63	RETRACTABLE SUBSIDY
Subsidy Cost $R^\wedge 3$	\$73.31				
$Q^\wedge=Q$			25.00		
P			10.00		
R			250		
Subsidy Cost R	\$50.00				

**INTERPRETATION**

M1 vs M2            Subsidy makes a difference, so higher subsidy rate, lower  $R^\wedge$  and higher ROV.

M2 vs M3            Retractable subsidy offers greatest incentive for early investment.

**AZ Solar Energy<sup>4</sup>**

Arizona is currently second in the nation in utility-scale electricity generation from solar energy. 25% of the energy consumed in AZ homes is for air conditioning, which (conveniently for solar energy) is mostly used during day light hours. AZ Renewable Environmental Standard requires 15% of the electricity consumed in 2025 to come from renewable energy resources. In 2013

<sup>4</sup> Source: U.S. Energy Information Administration, [www.eia.gov/state/AZ](http://www.eia.gov/state/AZ)



only 7.8% of the net electricity generation came from renewable resources, primarily from the Glen Canyon and Hoover Dams.

### **Tax Incentives for Solar Energy**

AZ offers at least five different tax incentives for solar energy. The Credit for Solar Energy Devices (Form 310) is for such devices installed in AZ residences. The Credit for Solar Hot Water Heating Plumbing Stub Outs and Electric Vehicle Recharge Outlets (Form 319) is for houses or dwelling units. The Credit for Solar Energy Devices-Commercial and Industrial Applications (Form 336) is for non-residential applications. The Renewable Energy Production Tax Credit (Form 343) is for electricity produced using a qualified energy resource such as solar. The Solar Liquid Fuel Credit (Form 344) is for expenses for R&D costs associated with solar liquid fuel. See [www.azdor.gov](http://www.azdor.gov).

The U.S. federal government currently offers a 30% investment tax credit (ITC) for the installation of certain solar power facilities until December 2016, which then declines to 10%. There are also rebates (solar renewable energy credits, SRECs) which are partially market related, accelerated depreciation benefits, and sometimes bonus depreciation. See Vivint Solar Provider LLC (recent IPO of a sponsor of investment funds which sells energy from solar systems to customers or directly leases the solar energy systems to customers), and Adkins and Paxson (2013).

### **Empirics**

From the AZ electricity price series per annum volatility is calculated as  $STDEV$  of  $LN(\text{Price } t/\text{Price } t-1)*SQRT(12)$ . The annualized price drift might be  $12*AVERAGE$  of  $LN(\text{Price } t/\text{Price } t-1)$ , or  $12*LN(\text{Price End}/\text{Price BEGIN})/\text{Months}(\text{END}-\text{BEGIN})$ .

## References

Adkins, R. and D. Paxson (2013), "The effect of tax depreciation on the stochastic replacement policy", *European Journal of Operational Research* 229: 155-164.

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Intergovernmental Panel on Climate Change. *Climate Change 2014: Mitigation of Climate Change* (2014).

## CASE QUESTIONS

1. Substituting your calculations of the AZ price drifts and volatility ( $\sigma_R$ ), what are the R thresholds for the three subsidy models?
2. What is the effect of reducing K through investment tax credits?
3. Which subsidy arrangement should Joe advocate that will please voters and not damage the state of AZ?
4. Which of the six policy variable values should Joe propose changing?